**Section 1.1:**

*Exercise 10:*

a) r ∧￢q

b) p ∧ q ∧ r

c) r → p

d) p∧￢q ∧ r

e) (p ∧ q) → r

f) r ↔ (q ∨ p)

*Exercise 28:*

*a-)*

|  |  |  |
| --- | --- | --- |
| *P* | *￢p* | *p→￢p* |
| T | F | F |
| F | T | T |

b-)

|  |  |  |
| --- | --- | --- |
| *P* | *￢p* | *p↔￢p* |
| T | F | F |
| F | T | F |

c-)

|  |  |  |  |
| --- | --- | --- | --- |
| *P* | *q* | *p ∨ q* | *p⊕ (p ∨ q)* |
| T | T | T | F |
| T | F | T | F |
| F | T | T | T |
| F | F | F | F |

d-)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *P* | *q* | *p ∨ q* | *p ∧ q* | *(p ∧ q )→ (p ∨ q)* |
| T | T | T | T | T |
| T | F | T | F | T |
| F | T | T | F | T |
| F | F | F | F | T |

e-)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *p* | *q* | *￢p* | *q→ ￢p* | *p↔ q* | *(q → ￢p) ↔ (p ↔ q)* |
| T | T | F | F | T | F |
| T | F | F | T | F | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

f-)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *P* | *q* | *￢q* | *p ↔ q* | *p↔ ￢q* | *(p ↔ q) ⊕ (p ↔ ￢q)* |
| T | T | F | T | F | T |
| T | F | T | F | T | T |
| F | T | F | F | T | T |
| F | F | T | T | F | T |

*Exercise 32:*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *p* | *q* | *r* | *p ∨ q* | *a-)*  *(p ∨ q) ∨ r* | *b-)*  *(p ∨ q) ∧ r* | *p ∧ q* | *c-)*  *(p ∧ q) ∨ r* | *d-)*  *(p ∧ q) ∧ r* | *￢r* | *e-)*  *(p ∨ q)∧￢r* | *f-)*  *(p ∧ q)∨￢r* |
| T | T | T | T | T | T | T | T | T | F | F | T |
| T | T | F | T | T | F | T | T | F | T | T | T |
| T | F | T | T | T | T | F | T | F | F | F | F |
| T | F | F | T | T | F | F | F | F | T | T | T |
| F | T | T | T | T | T | F | T | F | F | F | F |
| F | T | F | T | T | F | F | F | F | T | T | T |
| F | F | T | F | T | F | F | T | F | F | F | F |
| F | F | F | F | F | F | F | F | F | T | F | T |

*Exercise 50:*

We write these symbolically:

* u → ￢a
* a→ s
* ￢s→ ￢u

Note that we can make all the conclusions true by making a false, s true, and u false. Therefore if the user cannot access the file system, they can save new files, and the system is not being upgraded, then all the conditional statements are true. Thus the system is consistent.

**Section 1.2:**

*Exercise 10:*

a-)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *P* | *q* | *￢p* | *p ∨ q* | *￢p∧ (p ∨ q)* | *[￢p∧ (p ∨ q)] → q* |
| T | T | F | T | F | T |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | F | T | F | F | T |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *p* | *q* | *r* | *p→ q* | *q→ r* | *(p → q) ∧ (q → r)* | *p → r* | *b-) [(p → q) ∧ (q → r)] → (p → r)* | *p ∨ q* | *(p ∨ q) ∧ (p → r) ∧ (p → r)* | *d-) [(p ∨ q) ∧ (p → r) ∧ (p → r)] → r* |
| T | T | T | T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T | T | F | T |
| T | F | T | F | T | F | T | T | T | T | T |
| T | F | F | F | T | F | F | T | T | F | T |
| F | T | T | T | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T | T | F | T |
| F | F | T | T | T | T | T | T | F | F | T |
| F | F | F | T | T | T | T | T | F | F | T |

*c-)*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *P* | *q* | *p→ q* | *p∧ (p→ q)* | *[p∧ (p→ q)] →q* |
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

*Exercise 12:*

We argue directly by showing that if the hypothesis is true then so is the conclusion. An alternative approach which we show only for part a is to use the equivalences listed in the section and work symbolically.

a-) Assume the hypothesis is true then p is false.Since p∨ q is true,we conclude that q must be true.Here is a more algebraic solution:

[￢p∧(p∨ q)] → q ≡ ￢[￢p∧(p∨ q)]∨q (Table 7, line 1)

≡ ￢￢p∨￢(p∨ q)]∨q (De Morgan’s law)

≡ p∨￢(p∨ q)∨q (double negation)

≡(p∨ q)∨￢ (p∨ q) commutative and associative laws

≡ T negation law

b) We want to show that if the entire hypothesis is true, then the conclusion p → r is true. To do this we need to only show that if p is true, then r is true. Suppose p is true, then by we conclude that q is true. It now follows from the second part of the hypothesis that r is true as desired.

c) Assume the hypothesis is true. Then p is true, and since the second part of the hypothesis is true, we conclude that q is also true.

d) Assume the hypothesis is true. Since the first part of the hypothesis is true, we know that either p or q is true. If p is true, then the second part of the hypothesis tells us that r is true; similarly, if q is true, then the third part of the hypothesis tells us that r is true. In either case we conclude that r is true.

*Exercise 22:*

Suppose that (p → q) ∧ (p → r) is true. We want to show that p → (q ∧ r) is true, which means that we want to show that q ∧ r is true whenever p is true. If p is true, and since we know that p → q and p → r are true from our assumption, we can conclude that q is true and that r is true. So q ∧ r is true, as desired. Conversely, suppose that p → (q ∧ r) is true. We need to show that p → q is true and that p → r is true, which means that if p is true, then so are q and r. But this follows from p → (q ∧ r).

*Exercise 46:*

|  |  |  |
| --- | --- | --- |
| *p* | *q* | *p | q* |
| T | T | F |
| T | F | F |
| F | T | F |
| F | F | T |

**Section 1.3**

*Exercise 10:*

a) We take that this means that one student has all three animals: ∃x(C(x) ∧ D(x) ∧ F(x)).

b) ∀x(C(x) ∨ D(x) ∨ F(x))

c) ∃x(C(x) ∧ F(x)∧¬D(x))

d) ¬∃x(C(x) ∧ D(x) ∧ F(x)).

e) (∃xC(x))∧(∃xD(x))∧(∃xF(x)).

*Exercise 40:*

There are many ways to write these depending on what we use for predicates.

a)Let F(x) be “There is less than x megabytes free on the hard disk,” with the domain of discourse being positive numbers, and let W(x) be “User x is sent a warning message.” We have F(30) → ∀xW(x).

b) Let O(x) be “Directory x can be opened,” let C(x) be “File x can be closed,” and let E be the proposition

“System errors have been detected.” Then we have E → ((∀x￢O(x)) ∧ (∀x￢C(x))).

c) Let B be the proposition “The file system can be backed up,” and let L(x) be “User x is currently logged on.” Then we have (∃xL(x)) →￢B.

d) Let D(x) be “Product x can be delivered,” and M(x) be “The connection speed is at least x megabyts of memory available,” and S(x) be “The connection speed is at least x kilobits per second,” where the domain of discourse for the last two propositional functions are positive numbers. Then we have (￢M(8) ∧ S(56)) → D(video on demand)

**Section 1.4:**

*Exercise 10:*

a) ∀xF(x, Fred)

b) ∀yF(Evelyn, y)

c) ∀x∃yF(x, y)

d) ￢∃x∀yF(x, y)

e) ∀y∃xF(x, y)

f) ￢∃x(F(x, Fred) ∧ F(x, Jerry))

g) ∃y1∃y2(F(Nancy, y1) ∧ F(Nancy, y2) ∧ y1 \_= y2 ∧ ∀y(F(Nancy, y) → (y = y1 ∨ y = y2)))

h) ∃y(∀xF(x, y) ∧ ∀z(∀xF(x, z) → z = y)) i) ￢∃xF(x, x)

j) ∃x∃y(x \_= y ∧ F(x, y)∧∀z((F(x, z) ∧ z \_= x) → z = y))

*Exercise 16:*

P(s, c, m) will be the statement that student s has class standing c and is majoring in m. The variable

s ranges over students in the class, the variable c ranges over the four class standings, and the variable m ranges over all possible majors.

a)The proposition is ∃s∃mP(s, junior,m).It is true from the given information.

b) The proposition ∀s∃cP (s, c, computer science). It is false, since there are some math majors.

c) The proposition ∃s∃c∃m\_P(s, c,m) ∧ (c not = junior) ∧ (m not = mathematics). This is true, since there is a sophomore majoring in computer science.

d) The proposition is ∀s\_∃cP (s, c, computer science) ∨ ∃mP(s, sophomore,m). This is false, since there is a freshman mathematics major.

e) The proposition is ∃m∀c∃sP (s, c,m). This is false. It cannot be that m is mathematics, since there is no senior mathematics major, and it cannot be that m is computer science, since there is no freshman computer science major. Nor, of course, can m be any other major.

*Exercise 28*:

**a)** True (let *y* = *x*2 )

**b)** False (no such *y* exists if *x* is negative)

**c)** True (let *x* = 0)

**d)** False (The commutative law for addition always holds)

**e)** True (let *y* = 1*/x*)

**f)** False (the reciprocal of *y* depends on *y* —there is not one *x* that works for all *y* )

**g)** True (let *y* = 1*− x*)

**h)** False (this system of equations is inconsistent)

**i)** False (this system has only one solution; if *x* = 0, for example, then no *y* satisfies *y* = 2*∧−y* = 1)

**j)** True (let *z* = (*x* + *y*)*/*2)

*Exercise 40:*

**a)** There are many counterexamples. If x =2, then there is no y among the integers such that 2=1/y, since the only solution of the equation is y=1/2.Even if we were working in the domain of real numbers ,x=0 would provide a counterexample, since 0=1/y for no real number y.

**b)** We can rewrite*y^*2 *−x <* 100 as *y^*2 *<* 100 + *x*. Since squares can never be negative. No such y exists if x is say -200.

**c)** This is not true, since sixth powers are both squares and cubes. Trivial counterexamples would include

*x* = *y* = 0 and *x* = *y* = 1, but we can also take something like *x* = 27 and *y* = 9, since 27^2 = 3^6 = 9^3.

*Exercise 48:*

We need to show that each of these propositions implies the other. If ∀xP(x)∨∀xQ(x) is true.We need to show that ∀x∀y(P(x)∨Q(y)) is true. By our hypothesis, one of two things must be true. Either P is universally true, or Q is universally true. In the first case, ∀x∀y(P(x)∨Q(y)) is true, since the first expression in the disjunction is true, no matter what x and y are; and in the second case, ∀x∀y(P(x) ∨ Q(y)) is also true, since now the second expression in the disjunction is true, no matter what x and y are. Next we need to prove the converse. So suppose that ∀x∀y(P(x) ∨ Q(y)) is true. We want to show that ∀xP(x) ∨ ∀xQ(x) is true. If ∀xP(x) is true, then we are done. Otherwise, P(x0) must be false for some x0 in the domain of discourse. For this x0 , then, the hypothesis tells us that P(x0) ∨ Q(y) is true, no matter what y is. Since P(x0) is false, it must be the case that Q(y) is true for each y . In other words, ∀yQ(y) is true, or, to change the name of the meaningless quantified variable, ∀xQ(x) is true. This certainly implies that ∨xP(x)∨∀xQ(x) is true as desired.

**Section 1.5:**

*Exercise 6:*

Let *r* be the proposition “It rains,” let *f* be the proposition “It is foggy,” let *s* be the proposition “The sailing race will be held,” let *l* be the proposition “The lifesaving demonstration will go on,” and let *t* be the proposition “The trophy will be awarded.” We are given premises (*￢r ∨ ￢f*) *→* (*s ∧ l*), *s → t* , and *￢t*. We want to conclude *r* . We set up the proof in two columns, with reasons, as in Example 6. Note that it is valid to replace sub expressions by other expressions logically equivalent to them.

1. *￢t* Hypothesis

2. *s → t* Hypothesis

3. *￢s* Modus tollens using (1) and (2)

4. (*￢r ∨￢f*) *→* (*s ∧ l*) Hypothesis

5. (*￢*(*s ∧ l*)) *→￢* (*￢r ∨￢f*) Contrapositive of (4)

6. (*￢s∨￢l*) *→* (*r ∧ f*) De Morgan’s law and double negative

7. *￢s∨￢l* Addition, using (3)

8. *r ∧ f* Modus ponens using (6) and (7)

9. *r* Simplification using (8)

*Exercise 10:*

**a)** If we use modus tollens starting from the back, then we conclude that I am not sore. Another application of modus tollens then tells us that I did not play hockey.

**b)** We really can’t conclude anything specific here.

**c)** By universal instantiation, we conclude from the first conditional statement by modus ponens that dragonflies have six legs, and we conclude by modus tollens that spiders are not insects. We could say using existential generalization that, for example; there exists a non-six-legged creature that eats a six-legged creature, and that there exists a non-insect that eats an insect.

**d)** We can apply universal instantiation to the conditional statement and conclude that if Homer (respectively,Maggie) is a student, then he (she) has an Internet account. Now modus tollens tells us that Homer is not a student. There are no conclusions to be drawn about Maggie.

**e)** The first conditional statement is that if *x* is healthy to eat, then *x* does not taste good. Universal instantiation and modus ponens therefore tell us that tofu does not taste good. The third sentence says that if you eat *x*, then *x* tastes good. Therefore the fourth hypothesis already follows (by modus tollens) from the first three. No conclusions can be drawn about cheeseburgers from these statements.

**f)** By disjunctive syllogism, the first two hypotheses allow us to conclude that I am hallucinating. Therefore by modus ponens we know that I see elephants running down the road.

*Exercise 14:*

a) Let c(x) be “x is in this class,” let r(x) be “x owns a red convertible,” and let t(x) be “x has gotten

a speeding ticket.” We are given premises c(Linda), r(Linda), ∀x(r(x) → t(x)), and we want to conclude

∃x(c(x) ∧ t(x)).

1. ∀x(r(x) → t(x)) Hypothesis

2. r(Linda) → t(Linda) Universal instantiation using (1)

3. r(Linda) Hypothesis

4. t(Linda) Modus ponens using (2) and (3)

5. c(Linda) Hypothesis

6. c(Linda) ∧ t(Linda) Conjunction using (4) and (5)

7. ∃x(c(x) ∧ t(x)) Existential generalization using (6)

b) Let r(x) be “r is one of the five roommates listed,” let d(x) be “x has taken a course in discrete mathematics,” and let a(x) be “x can take a course in algorithms.” We are given premises ∀x(r(x) → d(x)) and ∀x(d(x) → a(x)), and we want to conclude ∀x(r(x) → a(x)). In what follows y represents an arbitrary

person.

1. ∀x(r(x) → d(x)) Hypothesis

2. r(y) → d(y) Universal instantiation using (1)

3. ∀x(d(x) → a(x)) Hypothesis

4. d(y) → a(y) Universal instantiation using (3)

5. r(y) → a(y) Hypothetical syllogism using (2) and (4)

6. ∀x(r(x) → a(x)) Universal generalization using (5)

c) Let s(x) be “x is a movie produced by Sayles,” let c(x) be “x is a movie about coal miners,”

w(x) be “movie x is wonderful.” We are given premises ∀x(s(x) → w(x)) and ∃x(s(x) ∧ c(x)), and we want

to conclude ∃x(c(x) ∧ w(x)). In our proof, y represents an unspecified particular movie.

1. ∃x(s(x) ∧ c(x)) Hypothesis

2. s(y) ∧ c(y) Existential instantiation using (1)

3. s(y) Simplification using (2)

4. ∀x(s(x) → w(x)) Hypothesis

5. s(y) → w(y) Universal instantiation using (4)

6. w(y) Modus ponens using (3) and (5)

7. c(y) Simplification using (2)

8. w(y) ∧ c(y) Conjunction using (6) and (7)

9. ∃x(c(x) ∧ w(x)) Existential generalization using (8)

d) Let c(x) be “x is in this class,” let f(x) be “x has been to France,” and let l(x) be “x has visited the Louvre.” We are given premises ∃x(c(x) ∧ f(x)), ∀x(f(x) → l(x)), and we want to conclude ∃x(c(x) ∧ l(x)).

In our proof, y represents an unspecified particular person.

1. ∃x(c(x) ∧ f(x)) Hypothesis

2. c(y) ∧ f(y) Existential instantiation using (1)

3. f(y) Simplification using (2)

4. c(y) Simplification using (2)

5. ∀x(f(x) → l(x)) Hypothesis

6. f(y) → l(y) Universal instantiation using (5)

7. l(y) Modus ponens using (3) and (6)

8. c(y) ∧ l(y) Conjunction using (4) and (7)

9. ∃x(c(x) ∧ l(x)) Existential generalization using (8)

*Exercise 26:*

We want to show that the conditional statement *P*(*a*) *→ R*(*a*) is true for all *a* in the domain; the desired conclusion then follows by universal generalization. Thus we want to show that if *P*(*a*) is true for a particular *a*, then *R*(*a*) is also true. For such an *a*, by universal modus ponens from the first premise we have *Q*(*a*), and then by universal modus ponens from the second premise we have *R*(*a*), as desired.